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Generalized Utility Describing Cooperation and Fairness

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Abstract

In this paper we show that for the case of two subjects the inequity-aversion utility coincides with the social-welfare utility that weights egoistic preferences, Hick optimality and maximin fairness. We also compare the generalizations of the inequity-aversion and social-welfare utilities to the case of more than two players and show that these two utilities are generalized in different ways. Additionally to that we propose a new way to generalize a two subjects utility to the case of more than two subjects. The new way of the generalization ensures that neither the cooperative nor the fairness terms will dominate as the size of the group grows. The proposed generalization also makes the utility function less sensitive to the payoff of one player as the size of the group grows. Finally, we introduce a second type of fairness to model decisions in the proposer-responder settings more accurately.

1 Introduction

In economic, sociology, game theory and biology models based on the assumption of egoistic subjects, maximizing their own benefit, were standard for a long period of time. However, recent studies demonstrated that these models cannot adequately explain many experimental findings. In many settings people choose actions that do not maximize their own payoff when those actions affect payoffs of other subjects. Many experimental evidence of this kind of behavior have recently inspired development of models of "social preferences" that assume that subjects are self-interested, but also are concerned about the payoffs of others. These models try to capture such phenomena as altruism, cooperation, notion of fairness and reciprocity. Among the models designed to capture the social (nonself-interested) behavior three classes can be distinguished: models that are based on distributional concerns [1, 2, 7, 8, 9, 13], models that are based on a concern of reciprocity [11, 15, 17, 5] and models that combine elements of both approaches [3, 4, 6]. The importance of the distributional and reciprocal concerns depends on the type of the situations that has to be modeled. There are cases in which the reciprocity cannot be neglected. Usually these are the cases in which a subject, called "responder", needs to make a decision given the decision of his/her co-player, called "proposer". In this case the kind/unkind actions of the proposer are usually reciprocated by a reward or punishment of the responder. To model the reciprocal behavior of the responder the following factors have to be take into account: the options available to the responder, the option proposed by the proposer, the options that could be proposed by the proposer and believes of the responder about the intentions of the proposer. There are also cases in which the reciprocal component of the concerns is absent. In particular for the person who makes the very first move in the game the attractiveness of the options depends solely on the payoffs associated with the option. In other words, in the this situation only the distributional concerns matter. Even for these simple cases the there is no consensus about what is the correct way to model preferences of the subjects. For example, in the literature several alternative models have been proposed. The "difference-aversion models" assume that subjects are motivated to increase their personal payoff and to reduce difference between their and others payoffs [9]. In contrast, the "social-welfare models" assume that subjects preferences consist of three components: (1) a component responsible for an egoistic behavior, (2) Hicks optimality component which is responsible for a cooperative behavior, and (3) a fairness component [3]. Although the inequity-aversion and social-welfare utilities are based on different ideas, it has been shown in the literature that for the case of two subjects they can be represented by the same simple expressions and the difference between the utilities is only because of different values of the parameters [3]. In this paper we consider the case of more than two subjects and show that for this case the inequity-aversion and social-welfare utilities are different. We consider the transition from the two-subject case to the case of more than two subjects in more detail and propose a new way to make this transition. For the case of more than two subjects, the proposed utility differs from the social-welfare and inequity-aversion utilities proposed in the literature. We argue that the proposed form of the utility has several advantages in comparison with the earlier proposed utilities. Moreover, we introduce additional fairness term. The purpose of this term is to model the fact that proposers can make different decisions depending on how many points he/she and responder get if the proposition is rejected. We believe that this additional fairness terms is important for an adequate description of the proposers behavior. The construction of the proposed utility function was driven by practical need to describe behavior of subjects in a game used in the MARS-500 experiment to monitor interpersonal relationships [10, 18, 19, 12, 16].

2 Results

2.1 Proposer-Responder Settings

Let us consider the proposer-responder setting implemented in many games and observed in many real life situations. In this setting the fist subject, called proposer, needs to choose and propose one of the available options. The responder, in his/her turn, can either accept or reject the proposition. If the proposition is accepted, it is implemented (i.e. the proposer and responder get the payoffs associated with the option). If the proposed option is rejected by the responder, a default option is implemented. The default option is also implemented if the proposer does not want to propose anything. It should be noted that the responder has the full information about the options that are available to the proposer. Both players also know what the default option is.

Let us consider the proposer-responder setting in more details. In the proposition phase the proposer chooses one option from a set of available options. Every option is characterized by two numbers: p and r that are payoffs of the proposer and responder, respectively. We assume that proposer (implicitly or explicitly) uses some utility function to estimate "attractiveness" of every option: $u = u(p, r)$. We also assume that attractiveness of options depend on the default payoffs of the proposer and responder p_0 and r_0 (payoffs that will be given to the proposer and responder in the case of no exchange): $u = u(p_0, r_0, p, r)$. In the case of the egoistic proposer, who cares only about his/her own payoff, the utility function is given by the following simple expressions: $u(p_0, r_0, p, r) = p$. In many cases (including ours) the above given egoistic utility function is too simple to describe behavior of proposer adequately. For example, the attractiveness of an option to the proposer might depend not only on the payoff provided by the option to the proposer but also on the probability that the considered option will be accepted by the responder and this probability, in its turn, depends on the payoff of the responder.

To describe preferences of subjects in simple test games different utility functions have been proposed. In this paper we will consider the simplest utilities based on clear qualitative ides. The first utility function that we will consider is the "inequity-aversion" utility [9]. This model assumes that subjects are motivated to increase their own benefit (egoistic preferences) and at the same time they are motivated to reduce difference between their own and others payoffs. The second utility that we will consider is called the "social-welfare" utility [3]. It assumes that the subjects are motivated to increase their own benefit as well as the social surplus, caring especially about helping those subjects who have a low payoff.

2.2 Inequity-Aversion and Social-Welfare Utilities for the Case of Two Subjects

Let us first consider the inequity-aversion utility in more details. For the case of two players it has the following simple form:

$$U_{ia}\left(p, r\right) = p - \alpha \max\left(r - p, 0\right) - \beta \max\left(p - r, 0\right). \tag{1}$$

This utility function has a simple interpretation. The first term represent egoistic preferences of the subjects. Additionally to that there are two terms that represent the fact that player do not like unequal distribution of the payoffs (inequity aversion). It is assumed that subjects do not like situations in which one of the players receives more than another one. It is also assumed that the degree of the dislike of an inequity depends not only on the amount of the inequity but also who is receiving more (I or my co-player). The last assumption is the reason why the utility function has two terms in addition to the egoistic one.

Let us now consider the social-welfare utility in more details. Charness and Rabin [3] have proposed to use the social-welfare utility function which, for the case of two subjects, has the following form:

$$U_{sw}(p,r) = (1-\lambda)p + \lambda\left[(1-\delta)(p+r) + \delta \min(p,r)\right]. \tag{2}$$

As we can see the social-welfare utility combines the egoistic preferences (the first term) with the social ones (the second term). If the parameters λ is equal to zero, the subject is absolutely egoistic. If λ is equal to one then the subject cares only about the social utility. The social term, in its turn, consist of the Hick optimality term and the maximin fairness. The Hick optimality corresponds to the maximization of the total benefit while the maximin fairness motivates the subject to increase the lowest payoff.

Let us now show that the inequity-aversion utility (1) coincide with the social-welfare utility (2) for the case of two subjects. For that let us find an explicit relation between the parameters of the inequity aversion (1) and social-welfare (2) utility functions written for two subjects. Before we start to search between the parameters of the two types of the utility functions we first need to ensure that both utility functions are "normalized" in the same way. In other words, we need to remove the ambiguity in the definition of a utility function that can rise from the fact that a decision maker is invariant with respect to the multiplication of the underlying utility by a positive number. For that we consider the both utilities for the $p = r$. In this case the inequity aversion utility is equal to p while the social-welfare utility is equal to $p(1+\lambda-\lambda\delta)$. This indicates that the two considered utility functions are different. We can easily see that this difference can be removed if slightly modify the original social-welfare utility function in the following way:

$$U_{sw}(p,r) = (1-\lambda)p + \lambda\left[\frac{1}{2}(1-\delta)(p+r) + \delta \min(p,r)\right] \tag{3}$$

Pay attention to the 1/2 in front of the term representing the Hick optimality (cooperative term). After this modification the social-welfare utility function will be equal to p for $p = r$. This modification can be done by redefinition of the parameters λ and δ in the expression. This modification does not change the meaning of the parameters. As before λ represents the portion of the egoistic and social terms in the utility. The λ equal to 1 represent a subject who cares only about the disinterested social utility (i.e. does not care about the personal benefit). In contrast λ equal to 0 represents the egoistic subject. The δ gives the balance between the Hick optimality and maximin fairness. The δ is equal to 1 means that the subject do not try to cooperate and cares only about the fairness of the outcome. The δ equal to 0, in contrast, represents cooperative subjects which do not care about the fairness of the outcome.

To demonstrate that the utilities (1) and (3) are the same we will consider them for two different cases: $r < p$ and $r > p$. For the $r > p$ the inequity aversion and the modified social welfare functions are equal two:

$$U_{ia}(p,r) = p(1+\alpha) - \alpha, r \tag{4}$$
$$U_{sw}(p,r) = \left[1 + \frac{\lambda}{2}(\delta - 1)\right] p - \frac{\lambda}{2}(\delta - 1) r. \tag{5}$$

For the case of $r < p$ we have the following expressions for the two utility functions:

$$U_{ia}(p,r) = p(1-\beta) + \beta r, \tag{6}$$
$$U_{sw}(p,r) = \left[1 + \frac{\lambda}{2}(\delta + 1)\right] p + \frac{\lambda}{2}(\delta + 1) r. \tag{7}$$

Just from the comparison of the expressions we can see that the two utility functions are identical and the following relation between the utility parameter should hold:

$$\alpha = \frac{\lambda}{2}(\delta - 1), \tag{8}$$
$$\beta = \frac{\lambda}{2}(\delta + 1). \tag{9}$$

Just for the record, we also give the inverse relation:

$$\lambda = \beta - \alpha, \tag{10}$$
$$\delta = \frac{\beta + \alpha}{\beta - \alpha}. \tag{11}$$

The relation between the inequity-aversion and the social-welfare utilities is easy see if we utilize the fact that both utilities are given by two linear functions one of which is valid for the region $r \geq p$ while another one is valid for the region $r \leq p$. It means that both utilities can be written in the following form:

$$U(p,r) = (1-\sigma)p + \sigma r \quad if \quad r \geq p, \tag{12}$$
$$U(p,r) = (1-\rho)p + \rho r \quad if \quad r \leq p, \tag{13}$$
$$\tag{14}$$

where σ and ρ some real constants. This way of writing of the utility functions was given in the work of Charness and Rabin [3]. In this form the utility function has a clear interpretation and the different types of the utility functions correspond to different ranges of the parameters σ and ρ. The inequity-aversion utility is given by the following inequalities: $\sigma < 0 < \rho < 1$. These social-welfare utility corresponds to the following range of the parameters: $1 \geq \rho \geq \sigma > 0$. Additionally to that we can mention the competitive preferences which are given by the following inequalities: $\sigma \leq \rho \leq 0$. These preferences mean that subject always prefers to do as well as possible in comparison to another subject. The visualization of the given relations between the parameters σ and ρ is given in

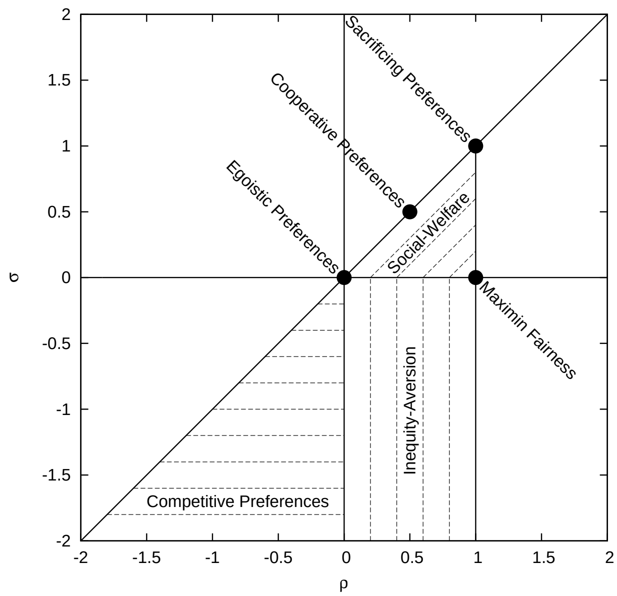

Fig. 1: Regions of the parameters σ and ρ corresponding to different classes of the utility functions

the figure 1. Additionally to the regions corresponding to the three mentioned classes of the utility functions the figure contains four points corresponding to the four special utility functions. The first shown utility function is the egoistic utility which describes subjects who care only about their own benefit. The cooperative utility corresponds to the subjects who put equal weights on each subjects payoff. In other words, these subjects try to maximize the total benefit. The sacrificing utility corresponds to the subject who cares only about the payoff of another subject. And finally the maximin fairness utility describes subjects who care only about the fairness of the outcome.

The figures 2-4 shows examples of the inequity-aversion, social-welfare and competitive utilities, respectively.

2.3 Inequity-Aversion and Social-Welfare Utilities for the Case of More than Two Subjects

The generalization of the earlier considered utility functions on the case of more than two subjects has been already studied by other researchers. In particular in the work of Fehr and Schmidt [9] which introduces the inequity aversion utility function (1) a form applicable to more than two players is also given:

$$U_i^{ia} = x_i - \frac{\alpha}{n-1}\sum_{\substack{j=1\\j\neq i}}^{n} \max\left(x_j - x_i, 0\right) - \frac{\beta}{n-1}\sum_{\substack{j=1\\j\neq i}}^{n} \max\left(x_i - x_j, 0\right) \qquad (15)$$

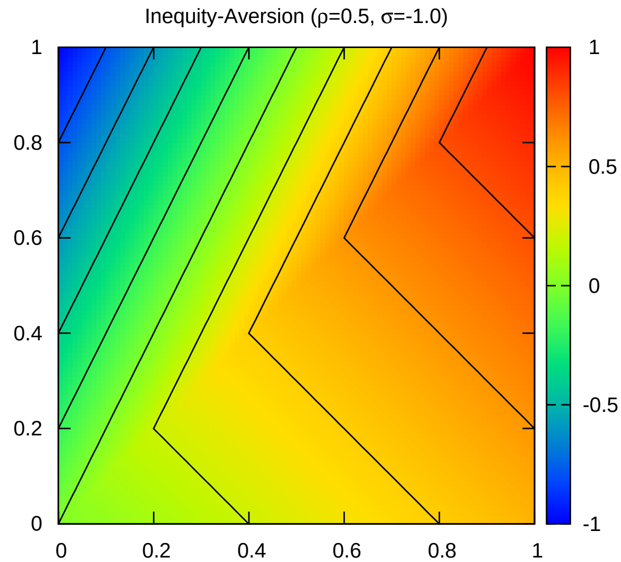

Fig. 2: Example of an inequity-aversion utility

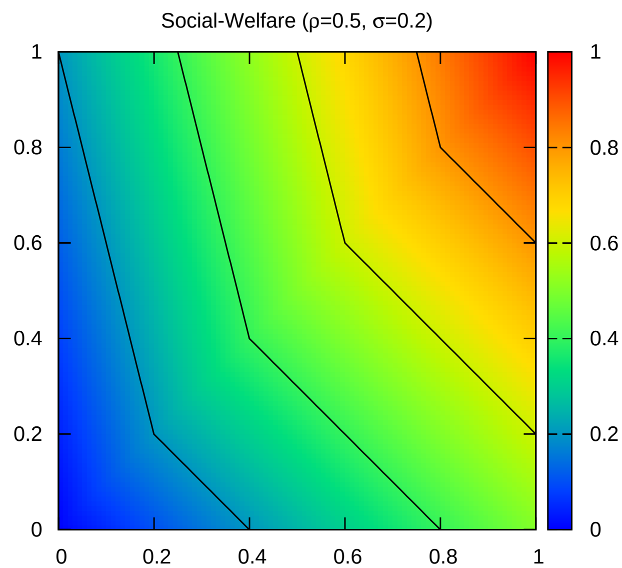

Fig. 3: Example of a social-welfare utility

Charness and Rabin [3], who introduce the social-welfare utility function, also give the generalization of the function on the case of more than two players:

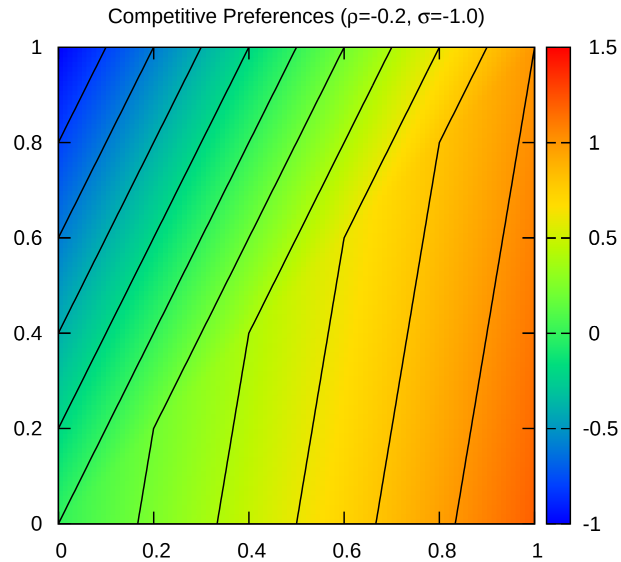

Fig. 4: Example of a competitive utility

$$U_i^{sw} = (1-\lambda)x_i + \lambda \left[(1-\delta) \sum_{i=1}^{n} x_i + \delta \min\left(x_1, x_2, \ldots, x_n\right) \right]. \tag{16}$$

As we have demonstrated, in the case of two subjects the inequity aversion and social-welfare utility functions coincide but the question if they are generalized to the case of many players in the same way arizes. We will demonstrate that these two utility functions are generalized in different ways. Moreover, we will provide the third way to make this generalization. We also argue why we think the generalization approach that we propose provides a more adequate description of the behavior of the subjects. First, we formalize the procedure that is used to generalize the inequity aversion function. Second, we apply this procedure to the social-welfare utility function for two subjects to make it more obvious that the many-subjects inequity aversion utility functions differs from the many-subjects social-welfare function. Third, we examine the logic behind the two ways to make the generalization to the many-subjects case and present a new way to make this generalization.

It can be easily demonstrated that the many-subjects inequity aversion utility function (15) can be obtained from the two-subjects utility function (1) in the following way:

$$U_{ia}^{(n)}\left(x_1, \ldots, x_i, \ldots, x_n\right) = \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} U_{ia}^{(2)}\left(x_i, x_j\right) \tag{17}$$

Let us apply this procedure to the modified social-welfare utility function (3):

$$\frac{1}{n-1}\sum_{\substack{j=1\\j\neq i}}^{n}(1-\lambda)x_i + \lambda\left[\frac{1}{2}\left(1-\delta\right)\left(x_i+x_j\right)+\delta\min\left(x_i,x_j\right)\right] =$$

$$\left(1-\lambda\right)x_i + \frac{\lambda\left(1-\delta\right)}{2}x_i + \frac{1}{n-1}\frac{\lambda\left(1-\delta\right)}{2}\sum_{\substack{j=1\\j\neq i}}^{n}x_j + \frac{\lambda\delta}{n-1}\sum_{\substack{j=1\\j\neq i}}^{n}\min\left(x_i,x_j\right). \quad (18)$$

As is evident, the structure of the above expression 18 is different from those of the social-welfare function written for more than two subjects (16), in spite on the fact that we started from the two-subjects social-welfare function (3). Let us consider the difference between the above expression and the many-subject social-welfare function (16) in more details. The first observation is that the above utility function, which is nothing but the inequity aversion utility written in a different way, does contain some terms that could be associated with the egoistic preferences, Hick optimality (cooperativeness) and the maximin utility. The first difference is that the cooperative term (the third one) in the equation 18 does not contain the contribution from the subject for whom the utility is given. Moreover, it is divided by the number of subject. In other words, the above inequity aversion utility function uses the average payoff of the subjects while the social-welfare utility uses the total payoff of the subject. To make the above expression closer to the social-welfare function we will remove the $n-1$ term from the expression. In this case the second term can go under the sum of the third term and we will get the same Hick optimality terms as in the social-welfare utility function:

$$\left(1-\lambda\right)x_i + \lambda\left[\frac{1}{2}\left(1-\delta\right)\sum_{j=1}^{n}x_j + \delta\sum_{\substack{j=1\\j\neq i}}^{n}\min\left(x_i,x_j\right)\right]. \quad (19)$$

Now the expression becomes more similar to the original social welfare utility function (16). The 1/2 coefficient in from of the Hick optimality term is there just because we used the modified version of the social-welfare function (3) instead of the original one (2).

2.4 A New Way to Generalize to the Case of More than Two Subjects

Besides that the only difference between the last expression and the original social-welfare function is the maximin fairness term. In the case of the social-welfare function the fairness is calculated as the minimal payoff in the group. In the above expressions the subject compares himself with all other subjects in the group and for every comparison the fairness is calculated and is added to the total fairness. This difference demonstrates that there could be different ways to calculate the total fairness or, in other words, there could be different ways to generalize the maximin fairness from the two-subjects case to the more than-two-subjects case. We think that using the term used in the social-welfare function has a fundamental problem. We will demonstrate that with the following example. Assume that a subject needs to choose between the following two situations. In the first situation 50 subjects get 11 points, other 50 subjects get 9 points and 1 subject gets 7 points. In the second situation 50 subjects get 12 points, other 50 subjects get 8 points and one subject gets 7 points. Transition from the first case to the second one seems to make the distribution of points less fair since the reach subjects start to get even more and pure subjects start to get even less. However, according to the expression used in the social-welfare function the fairness of both situations is the same (it is equal to 7, the minimal payoff in the group). To resolve this problem we propose to calculate the fairness of the payoffs distributions for all possible pairs of players and sum the values up:

$$\sum_{\substack{k,l=1 \\ k \neq l}}^{n} \min\left(x_k, x_l\right). \tag{20}$$

Additionally to that we have to decide how the weighting factor in front of this expression should depend on the number of the subjects in the group. The problem is that in the Hick optimality terms we have n summands while in the maximin that we have just introduced the number of terms is $n^2 - n$. As a result the fairness terms will dominate the cooperation term if n is larger enough. To resolve this problem we propose to calculate the average fairness. For that we divide the total fairness by number of terms under the sum $n(n-1)$. This quantity should not grow as n grows. Since the Hick optimality is proportional to n, we multiply the average fairness of the distribution by n. This way to combine the maximin fairness and Hick optimality ensures that none of this terms will dominate another one for large n. Summarizing we propose to use the following utility function for the case of more than to subjects:

$$U = (1-\lambda)\, x_i + \lambda \left[\frac{1}{2}(1-\delta) \sum_{j=1}^{n} x_j + \frac{1}{2} \frac{\delta}{n-1} \sum_{\substack{k,l=1 \\ k \neq l}}^{n} \min\left(x_i, x_j\right) \right]. \tag{21}$$

For the two players the above utility function is the same as the inequity aversion and the social-welfare utilities functions. For the case of more than two subjects all the three utilities functions are different.

2.5 The Second Type of Fairness

In the proposer-responder settings the decisions of the proposer (what option will be proposed) depend not only on the payoffs of the options but also on the payoffs of the default option [14]. By the default option we understand the option which will be implemented if proposer do not proposed anything or if the responder rejects the proposed option. In particular we can assume that the subjects care not only about the absolute payoffs associated with the options but also about the gain in the payoff that an option provides in addition to the default score. In other words we assume that the subjects perceive the payoffs of the default option as something that they already have since proposer and responder can always get the default option, if they want, independently on what are the actions of the co-player. As a consequence we could assume that subjects judge all other options by the amount of points that these options give in addition to what the subjects already have from the default option. We will call this amount as gain from the option. If we introduce the gain into the consideration, we can assume that it is treated in the same way as the absolute payoff of the option. In other words, we can assume that subjects might want to maximize their own gain or the total gain of all the subjects. However, utilities of this kind do not add anything new to the above considered utilities since a maximization o the gain is equivalent to the maximization of the absolute payoff. The same is valid for the total gain and total absolute payoff. In contrast, the maximum fairness of the gain is not identical to the maximin fairness calculated for the absolute score. In this way, the extended social-welfare utility function taking into account gains from the options can be written in the following way:

$$U_{4\omega}(p,r) = p + \omega_c(p+r) + \omega_f \min(p,r) + \omega_g \min(p-p_0, r-r0), \tag{22}$$

where ω_c, ω_f, and ω_g are weighting factors that represent importance of the cooperation and different kinds of fairness to the given player, respectively. The first term p capture the egoistic preferences. It has no coefficient (weighting factor) in front because the utility function is invariant with respect to a positive scaling factor and, as a consequence, we can always choose such a scaling factor that makes the coefficient in front of the egoistic term equal to zero. This normalization is convenient since in this case the importance of other factors will be given relatively to the importance of the own benefit of the player.

2.6 Removal of Redundancy from the Generalized Inequity-Aversion Utility

We have just introduced a generalization of the social-welfare utility function which captures two kinds of the fairness. Let us consider the question of the analogous generalization of the inequity-aversion function. The following expressions would be a straightforward way to make the generalization:

$$\begin{aligned} U_{gfs}(p,r) &= p - \alpha \max(r-p,0) - \beta \max(p-r,0) \\ &\quad -\alpha^* \max(\Delta r - \Delta p, 0) - \beta^* \max(\Delta p - \Delta r, 0)\,, \end{aligned} \tag{23}$$

where $\Delta p = p - p_0$ and $\Delta r = r - r_0$. We will call the above defined utility function as the generalize inequity aversion utility function.

In the case of the original inequity aversion utility function (1) the payoff space is split into two sub-spraces depending on who gets more points, proposer or responder. In each sub-space the utility function is just a liner function combining payoffs of the proposer and responder. In general case the linear functions from different regions differ from each other. In the case of the generalized utility aversion function the payoff space is split into three regions by two lines. The fist line is the same as in the case of the original inequity aversion utility function: $r = p$. The second line is parallel to the first one and goes through the default option: (p_0, r_0). The second line is given by the following equation: $r = r_0 + (p - p_0)$. In every region the generalized utility function is given by a linear function. As a consequence we need to use only three parameters to describe the generalized inequity aversion utility function (45). However, as we can see in the equation (45) there are four parameters. To remove the redundancy from the formula, let us consider the linear functions representing the utility function in different regions. There are four different cases: 1) $p > r$ and $\Delta p > \Delta r$, 2) $p > r$ and $\Delta p < \Delta r$, 3) $p < r$ and $\Delta p > \Delta r$, 4) $p < r$ and $\Delta p < \Delta r$. For these cases the utility function (45) reduces to:

$$U_{gia}(p,r) = p - (\beta + \beta^*)(p - r) - \beta^*(r_0 - p_0) \;\; if\; p > r \; and \; \Delta p > \Delta r, \quad (24)$$
$$U_{gia}(p,r) = p - (\beta - \alpha^*)(p - r) - \alpha^*(p_0 - r_0) \;\; if\; p > r \; and \; \Delta p < \Delta r, \quad (25)$$
$$U_{gia}(p,r) = p - (-\alpha + \beta^*)(p - r) - \beta^*(r_0 - p_0) \;\; if\; p < r \; and \; \Delta p > \Delta r, \quad (26)$$
$$U_{gia}(p,r) = p - (-\alpha - \alpha^*)(p - r) - \alpha^*(p_0 - r_0) \;\; if\; p < r \; and \; \Delta p < \Delta r. \quad (27)$$

The above set of equations can be rewritten in the following simplified form:

$$U_{gia}(p,r) = p - \alpha_{pp}(p - r) - c_{pp} \;\; if\; p > r \; and \; p > \Delta r, \quad (28)$$
$$U_{gia}(p,r) = p - \alpha_{pr}(p - r) - c_{pr} \;\; if\; p > r \; and \; p < \Delta r, \quad (29)$$
$$U_{gia}(p,r) = p - \alpha_{rp}(p - r) - c_{rp} \;\; if\; p < r \; and \; p > \Delta r, \quad (30)$$
$$U_{gia}(p,r) = p - \alpha_{rr}(p - r) - c_{rr} \;\; if\; p < r \; and \; p < \Delta r. \quad (31)$$

In the above equations the relation between the utility of the own benefit and benefit of the co-player is given by the coefficients α_{ij}.

$$\beta + \beta^* = \alpha_{pp}, \quad (32)$$
$$\beta - \alpha^* = \alpha_{pr}, \quad (33)$$
$$-\alpha + \beta^* = \alpha_{rp}, \quad (34)$$
$$-\alpha - \alpha^* = \alpha_{rr}, \quad (35)$$

It is easy to notice that the values of the coefficients α_{ij} are invariant with respect to the following transformation of the coefficients α, β, α^* and β^*:

$$\beta_{new} \rightarrow \beta_{old} + \gamma \quad (36)$$
$$\alpha_{new} \rightarrow \alpha_{old} - \gamma \quad (37)$$
$$\beta^*_{new} \rightarrow \beta^*_{old} - \gamma \quad (38)$$
$$\alpha^*_{new} \rightarrow \alpha^*_{old} + \gamma \quad (39)$$

So, to remove the ambiguity from the equations (i.e. to fix the γ) we can apply the following constrain on the coefficients:

$$\beta - \alpha = \beta^* - \alpha^* \quad (40)$$

If we have an initial set of the coefficients α_o, β_o, α_o^* and β_o^*, we can use transformation (32-35) to get the new values for the coefficients α_n, β_n, α_n^* and β_n^*. The parameter γ can be found in the following way:

$$\beta_n - \alpha_n = \beta_n^* - \alpha_n^* \tag{41}$$
$$(\beta_o + \gamma) - (\alpha_o - \gamma) = (\beta_o^* - \gamma) - (\alpha_o^* + \gamma) \tag{42}$$
$$(\beta_o - \alpha_o) + 2\gamma = (\beta_o^* - \alpha_o^*) - 2\gamma \tag{43}$$
$$\gamma = \frac{1}{4}[(\beta_o^* - \alpha_o^*) - (\beta_o - \alpha_o)] \tag{44}$$

On the next step we transform the expression (45) to explicitly utilize the above introduced relation between the utility parameters (40). First, we will use the following formula $\max(x, 0) = -\min(-x, 0)$.

$$U_{fs}^g(p, r) = p + \alpha \min(p - r, 0) - \beta \max(p - r, 0) \\ + \alpha^* \min(\Delta p - \Delta r, 0) - \beta^* \max(\Delta p - \Delta r, 0), \tag{45}$$

Second, we utilize the relation between the utility parameters to decrease their number from 4 to 3:

$$U_{3r}(p, r) = p - (\omega + f)\max(p - r, 0) - (\omega - f)\min(p - r, 0) - \\ -(\omega + g)\max(\Delta p - \Delta r, 0) - (\omega - g)\min(\Delta p - \Delta r, 0) \tag{46}$$

We will denote the utility function of this form as U_{3r} to indicate the fact that it splits the payoff space into 3 regions in which it is a linear function of the two parameters (p and r). The parameter f is responsible for the sensitivity of the subject to the inequity of the final benefit. If $f = 0$, then the terms containing f make the same contribution to the $p > r$ and $p < r$ regions. In other words, for a subject with $f = 0$ it does not matter what is larger his/her benefit (p) or benefit of his/her co-player (r).

2.7 Relation between the Generalized Social-Welfare and Inequity-Aversion Utilities

Let us now show that the utility function based on the idea of linear combination of four strategies (22) is identical to the generalized inequity aversion utility function (46). To find the relation between the two utility functions we will separately consider four different cases: 1) $p > r$ and $\Delta p > \Delta r$, 2) $p > r$ and $\Delta p < \Delta r$, 3) $p < r$ and $\Delta p > \Delta r$, 4) $p < r$ and $\Delta p < \Delta r$. For the first case ($p > r$ and $\Delta p > \Delta r$) we have:

$$U_{4\omega} = p(1 + \omega_c) + r(\omega_c + \omega_f + \omega_g) - \omega_g r_0, \tag{47}$$
$$U_{3r} = p(1 - 2\omega - f - g) + r(2\omega + f + g) + (\omega + g)(r_0 - p_0) \tag{48}$$

For the second case ($p > r$ and $\Delta p < \Delta r$) we have:

$$U_{4\omega} = p(1 + \omega_c + \omega_g) + r(\omega_c + \omega_f) - \omega_g p_0, \tag{49}$$
$$U_{3r} = p(1 - 2\omega - f + g) + r(2\omega + f - g) + (\omega - g)(r_0 - p_0) \tag{50}$$

For the third case ($p < r$ and $\Delta p > \Delta r$) we have:

$$U_{4\omega} = p(1 + \omega_c + \omega_f) + r(\omega_c + \omega_g) - \omega_g r_0, \quad (51)$$
$$U_{3r} = p(1 - 2\omega + f - g) + r(2\omega - f + g) + (\omega + g)(r_0 - p_0) \quad (52)$$

For the fourth ($p < r$ and $\Delta p < \Delta r$) case we have:

$$U_{4\omega} = p(1 + \omega_c + \omega_f + \omega_g) + r(\omega_c) - \omega_g p_0, \quad (53)$$
$$U_{3r} = p(1 - 2\omega + f + g) + r(2\omega - f - g) + (\omega - g)(r_0 - p_0) \quad (54)$$

By comparison of coefficients in front of p and r in the $U_{4\omega}$ and U_{3r} we will get the following set of 8 equations:

$$\alpha(1 + \omega_c) = 1 - 2\omega - f - g \quad (55)$$
$$\alpha(1 + \omega_c + \omega_g) = 1 - 2\omega - f + g \quad (56)$$
$$\alpha(1 + \omega_c + \omega_f) = 1 - 2\omega + f - g \quad (57)$$
$$\alpha(1 + \omega_c + \omega_f + \omega_g) = 1 - 2\omega + f + g \quad (58)$$

and

$$\alpha(\omega_c + \omega_f + \omega_g) = 2\omega + f + g \quad (59)$$
$$\alpha(\omega_c + \omega_f) = 2\omega + f - g \quad (60)$$
$$\alpha(\omega_c + \omega_g) = 2\omega - f + g \quad (61)$$
$$\alpha(\omega_c) = 2\omega - f - g \quad (62)$$
$$(63)$$

We have added a coefficient α to count for the fact that the two utility functions can use different normalization. If we add the first equation from the first subset of equations to the first equation from the second subset we get the following equation:

$$\alpha(1 + 2\omega_c + \omega_f + \omega_g) = 1 \quad (64)$$

The same equation can be obtained if we add the second, third of fourth equations from the two above subsystems. From the last equation we can easily get the scaling factor for the $U_{4\omega}$ utility based on its parameters.

$$\alpha = \frac{1}{1 + 2\omega_c + \omega_f + \omega_g} \quad (65)$$

After the scaling with the factor α the $U_{4\omega}$ will coincide with the U_{3r}:

$$\alpha U_{4\omega} = U_{3r}. \quad (66)$$

From the first equation of the fist subsystem it is easy to get ω_c. After ω_c is found we can use the second equation to find ω_g, and from the third equation we get ω_f. In this way we get the following solution:

$$\omega_c = \frac{1}{\alpha}(1 - \alpha - f - g - 2\omega) \tag{67}$$

$$\omega_g = \frac{2g}{\alpha} \tag{68}$$

$$\omega_f = \frac{2f}{\alpha} \tag{69}$$

2.8 The Case of More than Two Subjects with the Additional Fairness Term

The above proposed utility function models the fairness with respect to the final score. We would like to generalize the above function to include the second kind of fairness (the fairness with respect to the gain). To do so we could just add one more term to the utility function:

$$\sum_{\substack{k,l=1 \\ k \neq l}}^{n} \min\left(x_i - x_i^d, x_j - x_j^d\right), \tag{70}$$

where x_i^d denotes the default score of the player i. However, as we have seen before, in this way we will introduce the ambiguity to the utility function. In other words, different values of the utility parameters could give the same utility function. To resolve this problem we should take the two-subjects utility function with the removed ambiguity (46) and generalize it to the case of more than two players. To do so we have to formalize the generalization procedure. In other words, we have to find the procedure that transforms the modified social-welfare utility (3) to the desired many-subject utility functions (21). After we find this procedure, we can apply it to the two-subjects utility with removed ambiguity and, in this way, we should obtain the many-subject utility without ambiguity.

Let us first take the two-subject utility (3) and sum it up for all possible pares of subject and then divide by the number of the considered pares:

$$\frac{1}{n\,(n-1)} \sum_{\substack{k,l=1 \\ k \neq l}}^{n} \left\{(1-\lambda)x_k + \lambda \left[\frac{1}{2}\,(1-\delta)\,(x_k + x_l) + \delta \min\,(x_k, x_l)\right]\right\} \tag{71}$$

The above equation can be transformed to the following form:

$$(1-\lambda)\,x_i + \lambda \left[(1-\delta)\,\frac{1}{n} \sum_{j=1}^{n} x_j + \frac{\delta}{n\,(n-1)} \sum_{\substack{k,l=1 \\ k \neq l}}^{n} \min\,(x_k, x_l)\right] \tag{72}$$

If we compare the above equation (72) and the earlier proposed equation (21), we can see that the equation (21) can be obtained from the equation (72) if the following substitution $\lambda \to \lambda n/2$ before the social term is used. In other words, the generalization of the two-subject case (3) can be done if we replace λ by $n\lambda/2$ and average this two-subjects utility for all possible pare of subjects. Now we can apply this procedure to the two-subjects utility (46) which incorporates the two kinds of fairness and in which the ambiguity in the parameters is removed.

$$U_{3r}(p,r) = p - \frac{1}{2\,(n-1)} \sum_{\substack{k,l=1 \\ k \neq l}}^{n} \left[(\omega + f)\max(p - r, 0) + (\omega - f)\min(p - r, 0)\right]$$
$$-\frac{1}{2\,(n-1)} \sum_{\substack{k,l=1 \\ k \neq l}}^{n} \left[(\omega + g)\max(\Delta p - \Delta r, 0) + (\omega - g)\min(\Delta p - \Delta r, 0)\right] \quad (73)$$

3 Conclusions

In this work we have compared the inequity-aversion and social-welfare utility functions. We have demonstrated that, in spite on the fact that the two considered utility functions are identical for the case of two subject, their generalizations to the case of more than two subjects are different. The generalization procedures have been compared and analyzed. As a result, we have proposed a new generalization procedure which is different from the generalization procedures applied to the inequity-aversion and social-welfare utility functions. The proposed way to generate a utility functions for more than two subjects provide several advantages. First, the fairness of the payoff distributions is calculated by consideration of all possible pairs of subjects. This is different from the calculation of the fairness prescribed by the inequity-aversion functions in which only pairs containing the subjects making the decision are considered. Second, the importance of the social contribution to the utility functions, as compared to the egoistic contribution, grows as the size of the group grows. The inequity-aversion utility does not have this property. Third, the utility proposed for the case of more than two subjects becomes less sensitive to the payoff of a single subject as the size of the group grows. This is different from the social-welfare utility which could be very sensitive to the payoff of one player even for huge groups. Finally, we have proposed an additional maximin fairness term to the utility function to capture the fact that proposers can behave differently depending on how many points the default option gives to the proposer and responder. Summarizing, we have proposed a many-subjects utility functions which can explain egoistic, cooperative and fair behavior of different kinds in the proposer-responder setting.

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