

# Measuring Coordination in 2D Positioning Tasks

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**Abstract:** Several measures for coordination have been introduced recently in the human-factors literature. These measures try to capture the quality of the path followed during a two-dimensional (2D) or three-dimensional (3D) positioning task. Most of these measures are derived in a fairly ad hoc way, rather than being based on sound theoretical concepts. After analysing some of the existing measures, we formulate a list of requirements that seem essential for any measure of coordination. Based on these requirements we propose a new measure for coordination that abides by these requirements. We subsequently describe an experiment that was performed to gather subjective impressions of coordination for a number of 2D paths and compare the existing and newly proposed coordination measures with the obtained subjective rankings.

**Keywords:** Coordination, interaction techniques, allocation of control, evaluation methods, Human-Computer Interaction.

## 1 Introduction

Recent years have seen the proliferation of a wide variety of input devices into the users workspace. These devices range from 2D mice to many different kinds of 3D input devices. Given this growing diversity, it becomes increasingly more difficult to distinguish between these input devices solely based on traditional performance measures such as task completion time and accuracy (error). The problem is that these measures do not tell us much about how people actually handle devices and why performance differs across devices (MacKenzie, 2001). Some people propose that a more informative measure would be one that captures the quality of the trajectories executed by the user while performing positioning tasks with the input device (Zhai 1998; Masliah, 2000; Masliah, 2001). Coordination is a term usually used for a measure that conveys this information (Zhai 1998). The term coordination is used with the same intuitive meaning in disciplines as diverse as medical diagnosis and rehabilitation and athletics (Fischer, 1997). Despite this widespread interest in coordination, a broadly accepted definition of the term is still elusive (Kondraske, 2000). While different measures for

coordination have been introduced (Zhai 1998, Masliah, 2000; Fischer, 1997; Kondraske, 2000), they are mostly formulated in an ad hoc way, and are not based on any computational theory. Thus a measurement procedure for coordination that agrees with our intuitive understanding, and that is based on firm theoretical concepts, would be most useful in this context. A starting point for developing such a procedure is to analyze existing measures.

## 2 Existing Measures

Kondraske and Vasta (Kondraske, 2000) have proposed a Neuromotor Channel Capacity measure (NMCC) that is based on Fitts' law. Fitts' law expresses movement time MT in terms of the ratio A/W of target distance A over target size W in a 1D selection task as

$$MT = a + b \log_2 \left( \frac{2A}{W} \right)$$

The 'index of performance'  $IP=1/b$  (in bits/sec) is claimed to be a useful measure of the clinically more familiar term "Coordination".

In an earlier paper, Fischer and Kondraske (Fischer, 1997) have proposed an alternative quality measure that can be derived from the 3D trajectory

followed during the movement of a point-wise source. Their measure is a multiplicative combination of 4 dimensions of performance (DOP), i.e., speed, smoothness, accuracy and volume appropriateness. They compared their measure with scores provided by clinical experts and obtained a 75% correlation. Note that only coordination for 3 degrees of freedom (DOF) translations can be described by this measure. Measuring coordination in higher DOF tasks that also involve orientation does not currently fit within this framework.

Zhai and Milgram (Zhai 1998) have proposed the inefficiency measure to express the coordination while using 3D input devices with 6 DOF. In case of translation, their measure is defined as the ratio of the difference in length of the actual path and shortest path to the length of the shortest path. They have also extended this definition to rotation coordination. They point out themselves, that the inefficiency measure is as yet an untested definition of coordination, which may or may not agree with our intuitive understanding of coordination.

Masliah (Masliah 2000, 2001) also proposed an alternative measure of coordination, called the *m*-metric. The *m*-metric is the product of “the simultaneity of control” and “the efficiency of control”. The simultaneity of control attempts to quantify the amount of error (i.e., difference from the target position) reduced simultaneously in each DOF. The efficiency of control is a weighted average of the ratio of optimal path length over actual path length as expressed separately for each DOF.

From the description above it may be clear that most of the current measures are based on heuristics and have come about in a fairly ad hoc way. In the next section we formulate requirements for a coordination measure that seem reasonable, and check whether or not the proposed measures satisfy these requirements.

### 3 Requirements for a Coordination Measure

Most of the proposed measures have one aspect in common, i.e., they try to measure coordination based on the path followed by the interaction object. This seems a primary requirement for any coordination measure. In rehabilitation engineering (Fischer 1997; Kondraske, 2000), the coordination measure depended solely on the path itself (or only the end points), while the inefficiency measure and the *m*-metric compare the actual path to an optimal path.

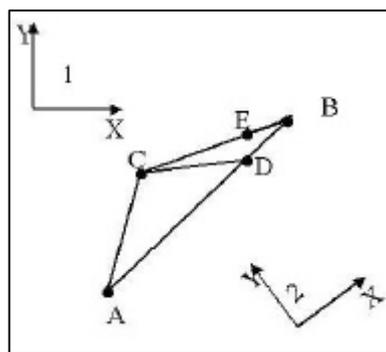


Figure 1: Example Paths (see text for discussion)

In his thesis, Masliah states that coordination measures should depend on the task (Masliah 2001, page 24). This means that if the task changes the same user action should potentially be attributed a different coordination score. This in turn implies that the measure should not only depend on the actual path, but also on the intended path towards the target position. This target position might be either stationary or moving. In the case of the inefficiency measure and the *m*-metric the optimal path does indeed depend on the target and starting positions. However, we argue that the optimal path should not be fixed, i.e., be determined by the start and end position, but instead should depend on the current position and target position. In Figure 1, the user deviates from the path ADB and arrives at C. Now that s/he is at C, the optimal path is to move along E to B, rather than going by means of CDB. However, measures such as the *m*-metric expect the user to return to D. We therefore propose that the coordination measure should be defined all along the path. Thus the measure should not be a single number, but a function of the path length. For example, if the user starts off with a bad coordination and then improves the coordination as s/he approaches the target, this should be deductible/observable from the measure.

In the above discussion it was implicitly assumed that positions are described by 2D or 3D coordinates. More generally, this position could refer to any set of DOF needed to fully describe the status of the input device. Just as position and rotation are aspects of a rigid body to which we can associate a coordination measure, scaling is another aspect that potentially should be considered, since the user might have to change the size of the original object to match it with the target object.

More theoretical requirements for trajectory measures are that they are independent of the coordinate system and sampling frequency. The

measure should provide the same score for the path ADB (see Figure 1) whether we calculate using the first or second coordinate system. For a rigid body in 3D space, there are 6 DOFs. If we take the translation and rotation about each axis as the 6 DOF and try to provide a coordination measure for each DOF separately then we will end up being dependent on how we define our coordinate system. This problem plagues the  $m$ -metric and the “smoothness” measure of DOP (Fischer 1997). For example, if we choose the first coordinate system to describe the path ADB in Figure 1, then the value for the  $m$ -metric is 1, while it changes to 0 if we choose the second coordinate system. Thus the same path can vary from being most coordinated ( $m = 1$ ) to least coordinated ( $m = 0$ ) by simply changing the coordinate system, which is clearly an unwanted feature. Similarly, increasing the number of data points along a trajectory should not change the value of the measure either.

Summarizing, we propose the following minimal requirements for a coordination measure:

*Req 1.* The measure should be trajectory based.

*Req 2.* The measure should compare the users actual path with an optimal path. The optimal path should depend on the current position and the target position.

*Req 3.* The measure should be applicable (or extendable) to rigid body positioning and rotation, and potentially also to scaling.

*Req 4.* The measure should be independent of the coordinate system and sampling rate.

Requirements? Measures ?	1	2	3	4
DOP (Fischer 1997)	V	X	X	V <sup>2</sup>
NMCC (Kondrask, 2000)	V	X	X	V
Inefficiency (Zhai 1998)	V	X*	V <sup>1</sup>	V
$m$ -metric (Masliah 2000, 2001)	V	X*	V <sup>1</sup>	X
V - meets requirement, X - does not meet requirement,				
* - The optimal path does not depend on current point 1 -No measure proposed for scaling 2 - The “smoothness” measure proposed is coordinate system dependent.				
<b>Table 1:</b> Comparison of existing coordination measures against formulated requirements.				

Despite the fact that these requirements seem fairly obvious, Table 1 demonstrates that none of the existing measures meet all of them.

## 4 Proposed Coordination Measure for 2D Positioning

Based on the set of requirements Req 1 to Req 4 we propose a measure for coordination that is based on the angle between the actual path taken by the user and the optimal path. Adhering to Req 2 the optimal path at any point is taken to be the shortest path from that point to the target point. In order to explain our new measure, we restrict ourselves here to 2D positional motion and refer to the Appendix for an extension of the measure to simultaneous translation and rotation. Extension to 3D and/or scaling is fairly straightforward but not discussed here.

In Figure 1, the angle between the two paths (AC) and (AB) determines the instantaneous coordination in moving from A to B. In the general case, where the paths (AB) and (AC) are curved, we would measure the angle between the tangents to these curves. The instantaneous coordination is hence derived from the cosine of the mutual angle between the tangent to the optimal path towards the target and the tangent to the actual path taken by the user, i.e.,

$$\cos \mathbf{q}_{m-1} = \frac{\langle (\vec{X}_m - \vec{X}_{m-1}), (\vec{X}_T - \vec{X}_{m-1}) \rangle}{d_{m-1,m} d_{m-1,T}},$$

$$d_{m-1,m} = \sqrt{\langle (\vec{X}_m - \vec{X}_{m-1}), (\vec{X}_m - \vec{X}_{m-1}) \rangle}$$

where  $\vec{X}_m$  is the  $m^{\text{th}}$  sampled position along the path,  $\vec{X}_T$  is the target position,  $\langle \dots \rangle$  is the inner product and  $d_{m-1,m}$  is the path length from the  $(m-1)^{\text{th}}$  sample position to the  $m^{\text{th}}$  sample position. This cosine can range from  $-1$  to  $1$ , with corresponding angle varying between  $0$  and  $180$  degrees. We may however consider angles that exceed a given maximum angle as being completely uncoordinated. This can be accomplished by deriving the instantaneous coordination from the cosine of the angle using the following expression

$$C_{m-1} = \max \left[ \frac{\cos \mathbf{q}_{m-1} - \cos \mathbf{q}_{\max}}{1 - \cos \mathbf{q}_{\max}}, 0 \right]$$

The above expression ensures that the instantaneous coordination varies from 1 to 0 for angles varying from 0 to  $q_{\max}$ .

In order to be able to compare this newly proposed coordination measure with other measures, we have to compress the instantaneous coordination measure as a function of path length into a single number. We propose the following Minkowski measure for this purpose

$$C = \left[ \frac{\sum_m C_{m-1}^P d_{m-1,m}}{\sum_m d_{m-1,m}} \right]^{\frac{1}{P}}$$

By varying  $P$  we can alter the emphasis on large coordination values. A value of  $P=1$  results in simply averaging the instantaneous coordination scores.

## 5 Subjective Evaluation

Since a widely accepted definition of the concept coordination is currently not available, we performed an experiment to find out if there is a shared understanding between subjects of the concept of coordination?

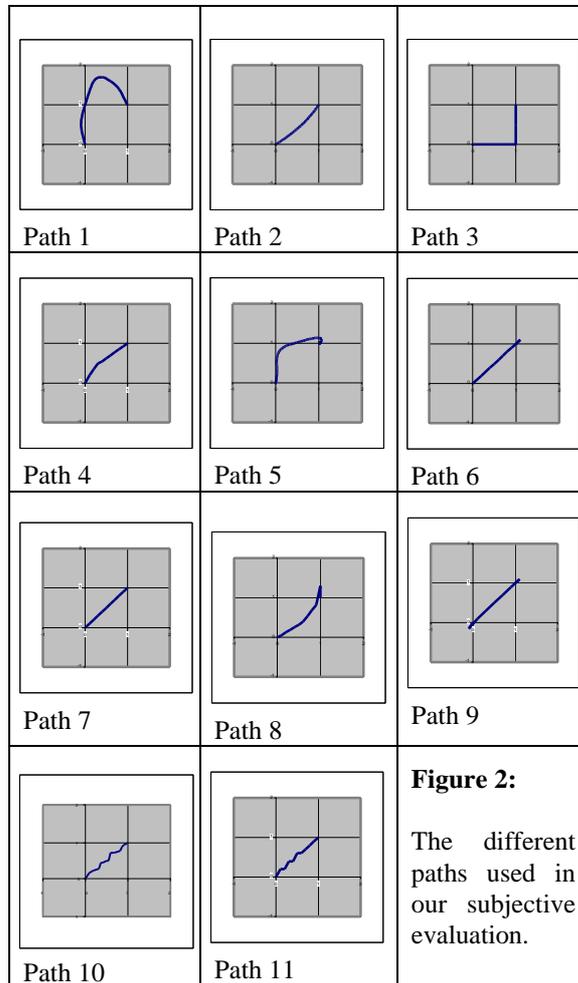
### 5.1 The Task

The subjects were shown 11 paths (see Figure 2) and were asked to rank them from most coordinated to least coordinated. The only explanation given was “When athletes run, one can tell whether their path was coordinated or not. This is based on people’s intuitive feeling for coordination. It is this feeling that we are trying to capture here.” No definition of coordination was given. The subjects had to arrange the 11 paths in the order of most coordinated to least coordinated. A blank grid was provided next to each path and the subjects were encouraged (but not obliged) to trace the path on this grid, before they ordered the different paths. After ordering the paths, they had to rate the difference in amount of coordination between successive pairs by means of an integer number between 0 and 3 (0: no difference, 1: slight difference, 2: clear difference, 3: large difference).

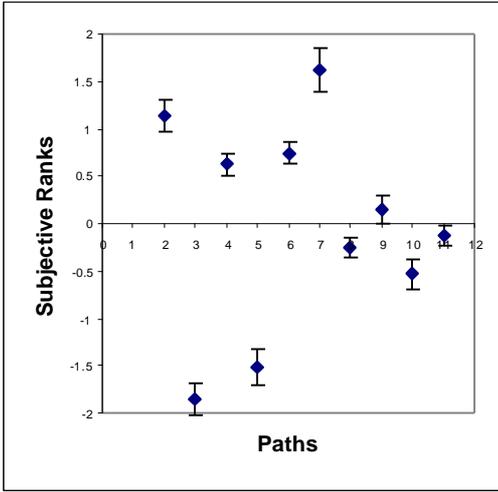
All paths started at A (0,0) and finished at B(1,1). In some cases the path overshoot the target B and returned to it (for example path 6), in other cases the movement was initially in the direction opposite to the location of the target (for example path 9).

## 5.2 Results

A total of 11 subjects participated in this evaluation. It took an average of 15 minutes to complete the task. The subjects were all in the age group of 20 to 35 years and consisted of 8 males and 3 females. None of the subjects actually traced the paths before judging them. All subjects rated the 7<sup>th</sup>



path as the most coordinated path and 10 of them rated the 1<sup>st</sup> path as the least coordinated one. The rank orders and the difference scores were analyzed by means of the multidimensional scaling program XGms (Martens, 2002). The resulting one-dimensional configuration, as well as the estimated 95% uncertainty intervals for the stimulus positions, are shown in Figure 3. These positions are determined on an interval scale, i.e., up to an arbitrary linear transformation. Path 1 was left out of the analysis as most users rated it consistently as the least coordinated path. The uncertainty intervals on the position of this stimulus were very large,



**Figure 3:** The one-dimensional coordination values with corresponding 95% uncertainty intervals, as derived from the subjective rankings by the XGms program.

signalling that this stimulus position could not be estimated reliably from the experimental data.

## 6 Results

The coordination values provided by the  $m$ -metric, the inefficiency measure and the proposed new metric were also calculated for each of the paths.

In case of the proposed new metric the scores were calculated for values of  $q_{\max}$  varying from 15 degrees to 45 degrees in steps of 5 degrees, using P values ranging from 0.1 to 1 in steps of 0.1. This resulted in 70 different scores per path for the proposed measure.

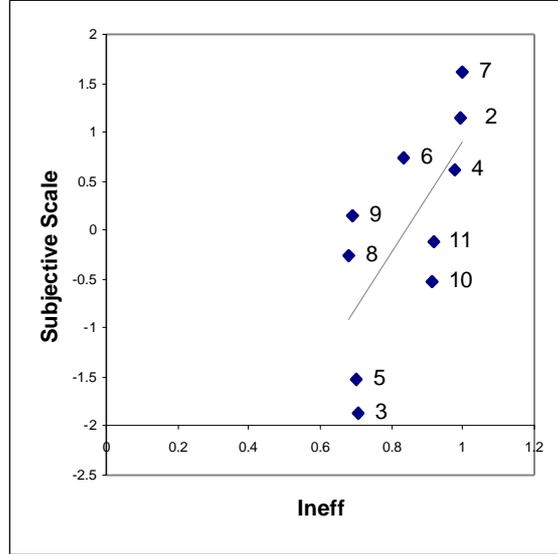
In case of the  $m$ -metric, a coordinate system had to be selected. The left-to-right direction was chosen to coincide with the + X-direction, while the bottom-to-top direction was chosen to be equal to the + Y direction.

Since the most coordinated path obtained the highest score in our experiment, we compared these scores with (1-inefficiency measure) for the inefficiency measure.

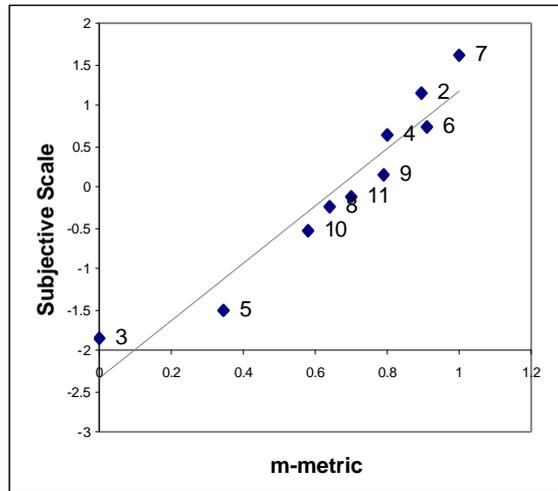
Figures 4, 5 and 6 show optimal linear regression lines between the objective measures, along the abscissa, and the subjective coordination scores along the ordinate. Figure 6 corresponds to the parameter choices  $P = 0.4$  and  $q_{\max} = 20$  for our new coordination measure that resulted in the highest linear correlation coefficient.

We compared the different measures in terms of their linear correlation coefficient with the subjective coordination scores, i.e.,

where  $n = 10$  (we only consider paths 2 to 11),  $\bar{X}$  denotes the mean for the  $X^{\text{th}}$  measure and  $S$  denotes the subjective coordination scores. Figure 7



**Figure 4:** Optimal linear regression between inefficiency measure and subjective coordination.



**Figure 5:** Optimal linear regression between  $m$ -metric and subjective coordination.

shows the distance,  $d = 1 - r_{xs}^2$ , from the subjective scale to the different measures, while Table 2 shows the linear correlation coefficients and the significance scores for the inefficiency measure, the  $m$ -metric, and the proposed measure with  $P = 0.4$  and

$q_{\max} = 20$  deg. The  $m$ -metric has the highest correlation with the subjective scores.

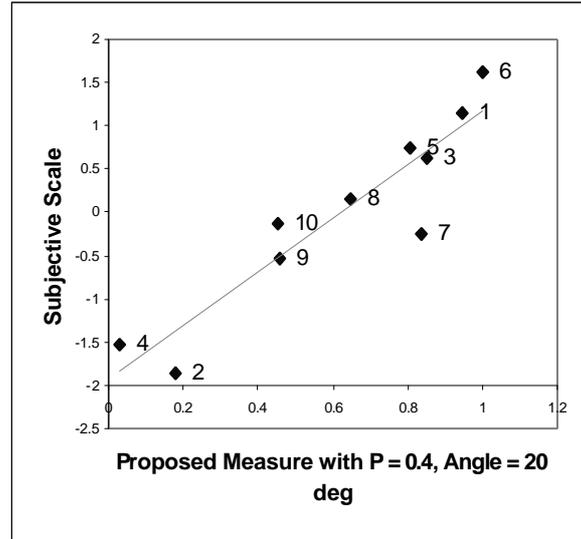
Inefficiency measure	$r = 0.6963$ , $F(1,8) = 7.528$ ( $p = 0.02531$ )
$m$ -metric	$r = 0.9560$ , $F(1,8) = 84.911$ ( $p = 0.00002$ )
Proposed metric $P = 0.4$ , $q_{\max} = 20$	$r = 0.9249$ , $F(1,8) = 47.340$ ( $p = 0.00013$ ).
<b>Table 2:</b> Correlation coefficients and significance scores for different measures.	

## 6 Discussion and Conclusions

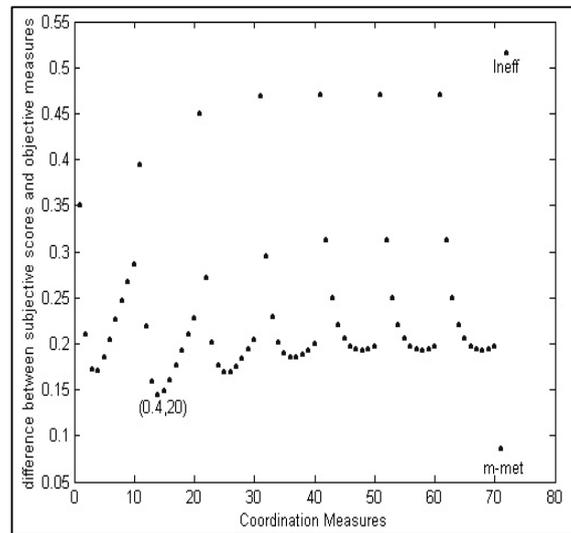
Table 2 indicates that the inefficiency measure correlates poorly with the subjective impression of coordination, as was already feared by the authors themselves (Zhai 98, page 325). This suggests that path length is certainly not the (only) criterion on which subjects base their impression of coordination.

The  $m$ -metric has the highest correlation with the subjective scores. The  $m$ -metric outperforms the proposed new measure, so that there must be properties of the paths that are captured by the  $m$ -metric and missed by the other measures. Therefore, a better understanding of coordination might be derived from the  $m$ -metric, provided that a computational theory can be established that more clearly explains the rationale behind the measurement procedure adopted by the  $m$ -metric. Another concern is that the  $m$ -metric performs well for the adopted coordinate system, but might fail completely if a rotated coordinate system were to be adopted. For example, if we rotate the original coordinate system by 45 degrees in the anti-clockwise direction, then the correlation coefficient falls to  $r_{xs} = 0.4132$ .

The proposed measure has the advantage that it captures a property of the path and exists at all points along the trajectory. In order to compare it against the subjective scores, we had to integrate the instantaneous coordination values into a single number. Alternative integration mechanisms may potentially result in a better correlation between objective and subjective coordination values. For example, if we introduce a weighting function which is inversely proportional to the integrated path length, into the averaging mechanism then the correlation coefficient increases to  $r_{xs} = 0.9413$ . This implies that points earlier in the path have a larger



**Figure 6:** Optimal linear regression between the proposed measure and subjective coordination.



**Figure 7:** Distance between the coordination measures and the subjective coordination scores.

effect on perceived coordination than points closer to the target.

One outcome of the experiment is that people do indeed share an intuitive understanding of the term “coordination”, since only one out of 11 subjects deviated significantly from the averaged coordination scores. This confirms that coordination can indeed be used as a performance measure to characterize different input devices based on how they allow users to trace a particular trajectory. This motivates further analysis of our intuitive understanding of coordination by means of more elaborate experiments.

## Appendix: Extending the proposed measure to rotational coordination

In case of simultaneously rotating and positioning a rigid body, the state of the body at sampling instant  $m$  can be described by its position and orientation, i.e.  $S_m = (\bar{X}_m, R_Q \cos j_m, R_Q \sin j_m)$ . The value  $R_Q$  parameterises the (fixed) size of the object and is a free parameter in the current discussion. In case of 2D positioning of a rigid body the state  $S_m$  is a 4D vector and is hence difficult to visualize. Figure 8 shows the case where positioning is restricted to 1D. In this case the state  $S_m$  reduces to a 3D vector. Rigid body motion then corresponds to moving the state vector on the surface of a cylinder with radius  $R_Q$  as depicted in Figure 8. The shortest path from A to B on this cylinder is the geodesic connecting A to B, which has the following mathematical expression (Thorpe, 1979)

where  $t \in [0,1]$ . When  $t = 0$ ,  $S_{A,B}(0) = S_A$  and when  $t$

$$S_{A,B}(t) = (\bar{X}_A(1-t) + \bar{X}_B t; R_Q \cos[(1-t)j_A + t j_B], R_Q \sin[(1-t)j_A + t j_B])$$

$= 1, S_{A,B}(1) = S_B.$

Following the arguments used in Section 4, the instantaneous coordination can be derived from the cosine of the mutual angle between the tangent to the optimal path towards the target and the tangent to the actual path taken by the user.

Thus if  $S'_{m,m-1}(0)$  is the derivative of  $S_{m,m-1}(t)$  at  $t=0$  then

$$\cos q_{m,m-1} = \frac{\langle S'_{m,m-1}(0), S'_{m-1,T}(0) \rangle_S}{\sqrt{\langle S'_{m,m-1}(0), S'_{m,m-1}(0) \rangle_S \langle S'_{m-1,T}(0), S'_{m-1,T}(0) \rangle_S}}$$

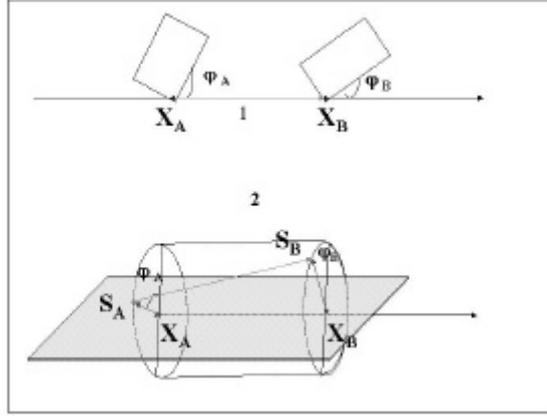
with state inner product

$$\langle S_1, S_2 \rangle_S = x_1 x_2 + y_1 y_2 + R_Q^2 \cos(j_1 - j_2).$$

This simplifies to

$$\cos q_{m,m-1} = \frac{\langle (\bar{X}_m - \bar{X}_{m-1}), (\bar{X}_T - \bar{X}_{m-1}) \rangle + R_Q^2 (j_m - j_{m-1})(j_T - j_{m-1})}{L_{m,m-1} L_{m-1,T}}$$

where  $L_{m,m-1} = \sqrt{d_{m,m-1}^2 + R_Q^2 (j_m - j_{m-1})^2}$  is the path length for the path from the  $(m-1)^{\text{th}}$  instant to the  $(m)^{\text{th}}$  instant.



**Figure 8:** 1 Object can be translated along a 1D axis (position  $X$ ) and rotated (angle  $j$ ); 2. 3D state space allows to simultaneously depict translation and rotation.

This equation extends the instantaneous coordination measure introduced in Section 4 to simultaneously measure for rotational and positional coordination. The arbitrary constant  $R_Q^2$  controls the emphasis on rotational or positional coordination. When  $R_Q^2 = 0$  the equation reduces to the one in section 4.

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